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WHITE PAPER

The Language of Mathematics:

Toward an Equitable Mathematics Pedagogy

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“Education, then, beyond all other devices of human origin, is the great equalizer of the conditions of men, the balance wheel of the social machinery.”

. . .

Horace Mann, 1848

Introduction

From the earliest days of public education in the United States, educators have believed in education's power as an equalizing force, an institution that would ensure equity of opportunity for the diverse population of students in American schools. Yet, for as long as we have been inspired by Horace Mann's vision, our history of segregation and exclusion reminds us that we have fallen short in practice. In 2019, 41% of fourth graders and 34% of eighth graders demonstrated proficiency in mathematics on the National Assessment of Educational Progress (NAEP). Yet only 20% of Black fourth graders and 28% of Hispanic fourth graders were proficient in mathematics, compared with only 14% of Black eighth graders and 20% of Hispanic eighth graders. Given these persistent inequities, how can education become "the great equalizer" that Mann envisioned? What, in short, is an equitable mathematics pedagogy?

Mathematical Competencies

To help think through this question, we will examine the **language of mathematics** and the role of language in math classrooms. This role, of course, has changed over time. Ravitch gives us this picture of math instruction in the 1890s:

> Some teachers used music to teach the alphabet and the multiplication tables…with students marching up and down the aisles of the classroom singing…"Five times five is twenty-five and five times six is thirty…" (Ravitch, 2001).

Such strategies for the rote learning of facts and algorithms once seemed like all we needed to do to teach mathematics. Yet, as educators in the twentieth century began to stress new mathematical competencies—such as solving novel problems, collaborating with peers, and sense-making—tension arose between the need for fluency with facts and algorithms versus focus on conceptual understanding. At the height of the Math Wars in the 1990s, one author defended teaching algorithms like this:

> Could these authors [who advocate against teaching algorithms] be unaware of the fact that the addition algorithm, like all other standard algorithms, contains mathematical reasoning that would ultimately enhance children's understanding of our decimal number system? Why not consider the alternative approach of teaching these algorithms properly before advocating their banishment from classrooms? (Wu, 1999).

In the twenty-first century, next-generation standards, including the Common Core State Standards for Mathematics (CCSSM) and the individual states' own mathematical frameworks, have tried to strike an appropriate balance between the need for fluency, skills, and conceptual understanding. Yet, even as most educators now agree that such a balance must be found, it can be bewildering to try to integrate into classroom practice all the aspects of math instruction that researchers now try to integrate.

To help educators navigate the multiple skills, capacities, and proficiencies demanded by the discipline, Moschkovich (2015) suggests we examine the multiple **competencies** required for what she terms **academic literacy in mathematics.** These include the following.

1. **Cognitive competencies,** including procedural fluency, conceptual understanding, meta-cognitive behaviors such as a growth mindset, and productive beliefs about the utility of mathematics.
2. **Cultural competencies** necessary for participating in the cultural practice of mathematics, including "problem solving, sense-making, reasoning, modeling, and looking for patterns, structure, or regularity" (Moschkovich, 2015, p. 1068). These competencies are articulated by frameworks such as the CCSS Standards for Mathematical Practice.
3. **Communicative competencies** necessary for communicating with others about mathematics, in spoken word, writing, and symbols, and in both the everyday register and the academic register.

Moschkovich (2015) names these competencies mathematical proficiency, mathematical practice, and mathematical discourse. Though Moschkovich focused on academic literacy in mathematics for English learners, we believe her framework is illuminating for all educators and all students.

In this paper, we use Moschkovich's schema as a starting point to explore the necessary role of language in mathematics classrooms. We examine, in turn, mathematical proficiency, practice, and discourse, and implications of each for classroom practice. We also consider the lens of special student populations. Through these investigations, we hope to articulate the necessary components of equitable mathematics pedagogy.

In defining mathematical proficiency, Moschkovich (2015) follows the **National Research Council** (2001),which suggests five strands of mathematical proficiency, including

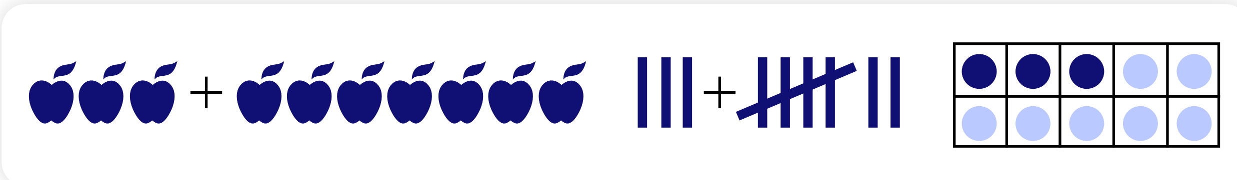
1. **conceptual understanding**—comprehension of mathematical concepts, operations, and relations;
2. **procedural fluency**—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
3. **strategic competence**—ability to formulate, represent, and solve mathematical problems;
4. **adaptive reasoning**—capacity for logical thought, reflection, explanation, and justification; and
5. **productive disposition**—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's efficacy (National Research Council, 2001).

Seen in this light, what Wu calls the "bogus dichotomy" between conceptual understanding and procedural fluency (Wu, 1999) begins to collapse. It's clear that we must inculcate in our students *comprehension* of mathematical operations and procedures as well as *fluency* in carrying them out.

Of course, it may be difficult to disentangle this dichotomy in practice. Must we not make a choice, for example, between using one model or multiple models? Between letting students discover their algorithm versus teaching a standard algorithm? We argue that the key to resolving this apparent dilemma is to understand how mathematical ideas are articulated through the curriculum, both within grades and across grades.

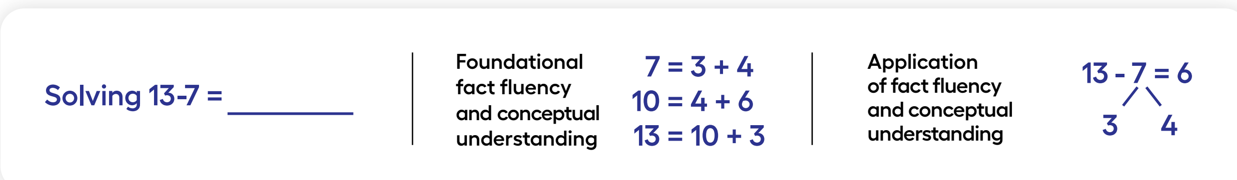
When students first learn to add and subtract within ten, they use concrete models, manipulatives, and counting strategies, such as counting the total, and later progress to “counting on” the second addend to the first addend (Common Core Standards Writing Team, 2018). Thus, students’ initial exposure to addition within and making 10 must be conceptually rich, and include ample time for exploration of multiple methods and representations.

FIGURE 1: *Multiple Methods and Representations for Adding to 10*



But mathematics is a hierarchical discipline. When students begin to add and subtract within 20, which requires composing and decomposing 10, they will much more easily be able to execute a “making 10” strategy if they have already developed fluency or even automaticity within 10 and a working understanding of place value. Such a student, who is working with 10 frames or another model to understand an equation like 13-7=6, will be well served by the ready knowledge that 7=3+4 and 13=10+3, which allows them to see that 13-7=13-3-4=10-4=6.

FIGURE 2: *Composing and Decomposing 10 for Adding and Subtracting within 20*



This knowledge of addition and subtraction within 20, in turn, must be proceduralized and then automated if students are to add and subtract whole numbers fluently. This is why the CCSSM (2010) requires students to “know from memory all sums of two one-digit numbers” by the end of second grade. As foundational skills are mastered, students free up working memory to devote to novel tasks. As Wu argues, “The automaticity in putting a skill to use frees up mental energy to focus on the more rigorous demands of a complicated problem” (Wu, 1999, p. 2).

This pattern of cognitive development reoccurs within grades and across grades. Moschkovich (2015) reminds us that:

> Research in cognitive science (Bransford, Brown, & Cocking, 1999) has shown that people remember procedures better, longer, and in more detail if they understand, actively organize, elaborate, and connect new knowledge to prior knowledge. In mathematics this means that in order to remember how to carry out computations, ELs will need to understand, elaborate, and organize procedures (Moschkovich, 2015).

If students develop a conceptual understanding of the algorithms they are learning, they are less likely to fall prey to what Stigler et al. call “conceptual atrophy” (Givvin et al., 2011). As the National Research Council (2001) observes, “A good conceptual understanding of place value in the base-10 system supports the development of fluency in multidigit computation. Such understanding also supports simplified but accurate mental arithmetic and more flexible ways of dealing with numbers than many students ultimately achieve” (National Research Council, 2001, p. 121).

We are not suggesting that students’ study of mathematics should be limited to the understanding, however conceptually rich, of algorithms, nor only to the study of routine problems. Routine problems are “problems that the learner knows how to solve based on past experience” (National Research Council, 2001, p. 126). If students are only exposed to problems for which they have already learned a method of solution, they will come to adopt negative beliefs about the power, interest, and applicability of mathematics. For example, “There is only one correct way to solve any mathematics problem—usually the rule the teacher has most recently demonstrated to the class” or “Mathematics problems have one and only one right answer” (Schoenfeld, 2017 p. 27). If solving a math problem means applying an algorithm, then there is little sense in engaging in productive struggle to find a novel solution; one knows the algorithm or one doesn’t.

Students begin to see beyond this limiting view—math as the mere application of algorithms—through practice in solving nonroutine problems. “Nonroutine problems require productive thinking because the learner needs to invent a way to understand and solve the problem” (National Research Council, 2001, p. 126). This invention of ways of thinking about and solving problems is not only a cognitive and metacognitive practice, but a cultural practice, best engaged in with peers and the guidance of a teacher. Perhaps this is why nonroutine problems have been called “group-worthy tasks.” We now turn to these cultural practices.

Mathematical Practices

So far, we have considered cognitive mathematical capacities, such as procedural fluency, conceptual understanding, and adaptive reasoning. Yet, these skills are applied by learners in a cultural context.

As Moschkovich explains:

> The five strands of mathematical proficiency [discussed above] provide a **cognitive** account of mathematical activity focused on knowledge, metacognition, and beliefs. From a **sociocultural** perspective, mathematics students are not only acquiring mathematical knowledge, they are also learning to participate in valued mathematical practices (Moschkovich, 2004, 2007, 2013a, 2013b). Some of these practices include problem solving, sense-making, reasoning, modeling, and looking for patterns, structure, or regularity (Moschkovich 2015, emphasis added).

The discipline of mathematics includes several overlapping bodies of practitioners, including students, teachers, and research mathematicians. Each subculture has its own body of practices, yet they contribute to a shared mathematical culture. The mathematical practices that best exemplify this culture are given in such documents as the National Council of Teachers of Mathematics Standards (2000) and the Standards for Mathematical Practice.

FIGURE 3: *Standards for Mathematical Practice (2010)*

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

What are the signifiers of these cultural practices and behaviors in the context of a particular mathematical task? To investigate this question, Schoenfeld (2017) gave novel, nonroutine mathematical problems to both university undergraduate students and to professional mathematicians, and documented the approach of members of each group. The undergraduate students "read the problem, quickly chose an approach to it, and pursued that approach. They kept working on it, despite clear evidence that they were not making progress, for the full 20 minutes allocated for the problem session" (Schoenfeld, 2017, p. 24). But the mathematicians, expert solvers encountering a novel problem, took a much different approach. They spent time planning, analyzing the problem, and verifying their solutions. While solving, they made metacognitive remarks, such as: "'Hmm, I don't know exactly where to start here' (followed by two minutes of analyzing the problem) or 'OK. All I need to be able to do is [a particular technique] and I'm done'" (Schoenfeld, 2017, p. 24).

Crucially, students can learn the cultural practices employed by expert solvers, but they must be explicitly taught. Schoenfeld had his students work in groups and guided with questions such as:

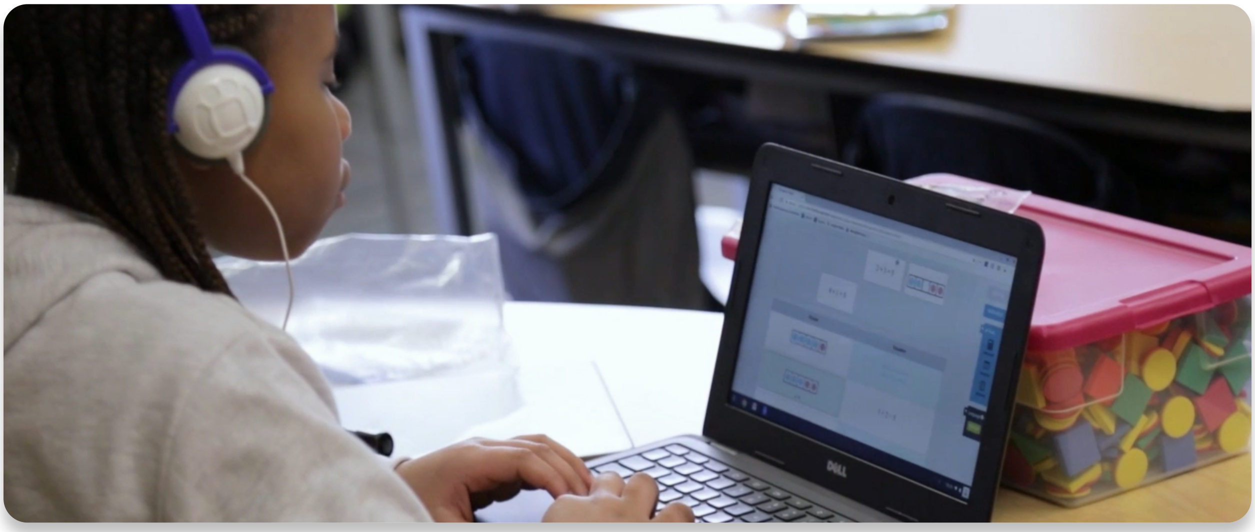
- What (exactly) are you doing? (Can you describe it precisely?)
- Why are you doing it? (How does it fit into the solution?)
- How does it help you? (What will you do with the outcome when you obtain it?) (Schoenfeld, 2017, p. 24)

Schoenfeld notes that at the beginning of the course, students weren't able to answer these questions; the ability to reflect on the problem-solving process developed only over time. Other research confirms that to induct students into the cultural practice of mathematical problem solving, "we must find ways of teaching math that encourage all students to participate in conversations about math problems…Participation turns out to be the crucial word here: students do not learn unless they contribute actively" (Featherstone et al., 2011, p. 29). By the end of a semester's explicit practice, Schoenfeld's students were solving problems with an approach more like that of an expert solver.

We wish to draw two conclusions from Schoenfeld's (2017) research:

1. Students can develop the set of cultural, cognitive, and metacognitive competencies that characterize mathematical problem solving as professional mathematicians practice it, but only with explicit instruction, participation, and practice.
2. These mathematical practices, and the strategies teachers must use to develop them, are necessarily couched in language and mediated through discourse.

We now consider the proper role of academic mathematical discourse.



Mathematical Discourse

Mathematical discourse, articulated through academic mathematical language, is the linchpin that holds together the strands of mathematical proficiency and practice, which we have discussed. Mathematical discourse involves not only oral and written text, but also multiple modes, representations (gestures, objects, drawings, tables, graphs, symbols, etc.), and registers (school mathematical language, home languages, and the everyday register). These registers include academic language, the language used in schooling for learning, which is very different from the language registers students use outside of school (Schleppegrell, 2007). Students' prior knowledge of language typically comes from what they learn at home and within their communities, sometimes referred to as *everyday language* (Simpson & Cole, 2015). Students' everyday language is influenced by social and cultural factors, and is often not academic in nature. On the other hand, "academic language is the set of words, grammar, and organizational strategies used to describe complex ideas, higher-order thinking processes, and abstract concepts" (Zwiers, 2008, p. 20). Academic mathematical language also includes technical vocabulary utilized by expert users, such as mathematicians.

Yet, we should not conclude that mathematical discourse, even among mathematicians, includes only technical vocabulary, or that the backgrounds, experiences, and language registers that students bring to the classroom are not an asset for learning. As Moschkovich explains:

> Academic mathematical discourse is not principally about formal or technical vocabulary (Moschkovich, 2007). The mathematics register is a complex construct that includes styles of meaning, modes of argument, and mathematical practices and has several dimensions such as the concepts involved, how mathematical discourse positions students, and how mathematics texts are organized…Textbook definitions and formal ways of talking are only one aspect of school mathematical discourse. In classrooms students use multiple resources, including everyday registers and experiences, to make sense of mathematics (Moschkovich 2015).

Students do not necessarily come to mathematics classrooms ready to use the language of mathematics when learning. Specifically, students need to be taught how to use the oral and written language of mathematics in classroom tasks, including academic vocabulary, styles of meaning, modes of argument, mathematical practices, and more (Moschkovich 2015). Mathematical discourse involves describing patterns, making generalizations, and using representations to support claims (Moschkovich, 2012). In order to engage in the mathematical process standards, such as constructing viable arguments and critiquing the reasoning of others, students need opportunities to practice with mathematical concepts, and to thoughtfully present, connect, and generalize mathematical ideas of their own and of others, both in writing and within the context of classroom and peer dialogues.

Students may initially be reticent to challenge their peers, to disagree, and to engage in academic conversation. Teachers need to help students understand the associated norms and cultural practices of mathematical discourse. One strategy teachers might employ is to model explicit "talk moves" students can use when discussing math with their peers. For example, students might learn to say "I disagree because" or "I challenge you," followed by a competing claim when they have a different point of view. Zwiers and Crawford (2011) detail a number of such strategies, including building on or challenging a partner's idea, elaborating and clarifying, supporting ideas with examples, paraphrasing, and synthesizing conversation points.

Mathematical Vocabulary

We have argued that acquiring academic *literacy* in mathematics cannot be understood primarily as a process of acquiring mathematical or symbolic *vocabulary*. This is not to say that vocabulary instruction should be absent from mathematics classrooms. "Vocabulary instruction is as important to math comprehension as it is to reading comprehension," and is still central to learning to read, write, speak, listen to, and make sense of mathematics (Bruun et al., 2015, p. 532; Moschkovich, 2015; Roberts & Truxaw, 2013). Further, mathematics research outlines a more complex view of mathematical language that includes both general and specialized vocabulary—new words and new meanings to known words—and an inclusion of the symbolic language in mathematics (Moschkovich, 2015; Schleppegrell, 2007). Examples of general academic words used in mathematics are: "describe," "summarize," "compare," and "evaluate." These general academic words can be associated with mathematical processes and thinking. Specialized mathematical vocabulary includes words like "rhombus," "circumference," "absolute value," and "polygon." Within math lessons, teachers should incorporate both general and specialized vocabulary associated with specific concepts.

The symbolic language in mathematics is specific to the discipline and presents particular challenges for students in understanding and communicating about mathematical concepts. "Symbolic representations, inherent within the technical language of mathematics, are at odds with students' everyday language and sign system" (Simpson & Cole, 2015, p. 378). For example, when learning to compare quantities, students use math-specific language "greater than" and "less than" and symbols that represent these terms. Early in elementary grades, students must process the words "greater than" and "less than," and acquire proficiency and fluency in reading symbols.

FIGURE 4: *Challenges with math vocabulary*

11 Reasons Why Learning the Language of Math is Challenging for Students

1. Meanings are context dependent (e.g., foot as in 12 inches vs. the foot of the bed).
2. Mathematical meanings are more precise (product as a multiplication solution vs. the product of a company).
3. Terms are specific to mathematical contexts (e.g., rhombus, equilateral, prime numbers).
4. Words have multiple meanings (side of a triangle vs. side of a cube).
5. Discipline-specific meanings (division in math vs. a division of a company).
6. Homonyms with everyday words (pi vs. pie).
7. Related but different words (radius and diameter).
8. Specific challenges with translated words (e.g., mesa vs. table).
9. Concepts can be expressed in multiple ways (e.g., two times three, two groups of three, three plus three).
10. Irregularities in spelling.
11. Students and teachers may adopt informal terms instead of mathematical terms (e.g., bigger is used for greater than).

(Riccomini et al., 2015, p. 238)

As illustrated in Figure 4, mathematical vocabulary can be particularly difficult for students to learn because students do not come to school knowing the vocabulary needed to engage with the language of mathematics. Vocabulary instruction in mathematics should utilize effective methods for teaching vocabulary in other disciplines. In general, vocabulary instruction should focus initially on promoting understanding and storing word meaning in long-term memory. Second, after students understand concepts and terms, the focus of vocabulary instruction should be on fluency and maintaining learning over time. Finally, students should be explicitly taught how to use mathematical vocabulary to explain and justify mathematical concepts and relationships (Riccomini et al., 2015). These overarching goals can be integrated into specific lessons. Vocabulary instruction within lessons should

- be explicit in stating word meaning and modeling how to use specific terms;
- stimulate and connect prior knowledge with new learning;
- provide opportunities for repetition to solidify learning;
- incorporate differentiation to account for students' instructional levels; and
- leverage cooperative learning to provide opportunities for students to use new vocabulary in talking about and doing math (Riccomini et al., 2015).

Attention to vocabulary instruction must take place in the context of a classroom focused on the broader goal of teaching mathematical discourse. We consider the following example from Moschkovich:

> For example, we can contrast the claim "Multiplication makes bigger," which is not precise, with the claim "Multiplication makes the result bigger, only when you multiply by a positive number greater than 1." When contrasting the two claims, notice that (1) precision does not lie in the individual words used and (2) the words used in the second claim are not more formal mathematical words. Instead, the precision of the second claim lies in specifying when the claim is true. In a classroom, if a teachers' response to the first claim focused on precision at the word level, a follow up question might be to ask a student to use a more formal word for "bigger." In contrast, if a teacher was focusing on precision at the discourse level, a follow up question would be "When does multiplication make a result bigger?" (Moschkovich, 2015, p. 48).

Teachers must strike a balance between the need to develop formal academic vocabulary and then to develop the discourse that uses that vocabulary, as well as an everyday language and other assets that students bring to the classroom.

Special Populations of Mathematics Learners

Early Elementary Education

The mathematics classroom is diverse, yet all students are held to the same mathematical standards, including the process standards. To better engage younger students with the language of mathematics, use sportscasting through talking aloud using self-talk, parallel talk, and reflective speech to highlight mathematical concepts. Researchers also suggest creating a supportive environment intentionally designed to foster mathematical activities and thinking through play. For example, setting up a play area with items of different shapes, and guiding student conversations to discuss numbers, make comparisons, talk about size or weights, patterns, changes of objects over time, or sorting by attributes of shapes (Luckenbill, 2018).

English Learners

Mathematics is linguistically complex, and includes multiple language modalities and specialized vocabulary. For students who speak English as a second language, researchers recommend using word walls and graphic organizers to help students develop their mathematics vocabulary as they help students relate to the content, and can be used as references throughout mathematics problem solving (Bay-Williams & Livers, 2009; Celedón-Pattichis & Ramirez, 2012; Roberts & Truxaw, 2013; Thompson & Rubenstein, 2000). Additionally, teachers should ask open-ended questions of all students, scaffolding English learners with close-ended questions, if needed. The intentional sequencing of close-ended questions can help a student work through mathematical problems (Banse, et al., 2016).

Students with Disabilities

Students with disabilities and low-achieving students may encounter difficulty engaging in mathematical discourse. Specific disabling conditions such as learning disabilities (in mathematics and reading) and speech or language impairments impact students' ability to process, understand, and utilize the language of math (Individual with Disabilities Education Act, 2004). Additionally, research indicates that students with math disabilities have deficits in their ability to solve computations and application problems in mathematics (Geary, 2004). In classroom settings, students with disabilities and low achieving students may be excluded from engaging in rich mathematical discourse because of limited verbal reasoning ability and cognitive load associated with discourse activities and curricular materials (Griffin et al., 2013).

To support students with disabilities and low-achieving students, mathematics instruction should be explicit and systematic, and include models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent, cumulative review. Teachers should explicitly teach students with disabilities how to solve word problems with common underlying structures. Intervention materials for students with disabilities should include opportunities for students to work with visual representations of mathematical ideas and develop fluency with symbolic representations. Additionally, students with disabilities benefit from regular practice in building fluency with math facts (Gersten et al., 2009).

Conclusion

The goal of college and career readiness standards is to elevate all students' learning and equitably prepare all students for 21st-century careers. These standards ask for a shift in mathematics classrooms, highlighted by the integration of the mathematical process standards, which require students to more deeply engage with mathematical concepts and the language of mathematics. The National Council of Teachers of Mathematics defines these outcomes for equitable instruction in mathematics:

> Acknowledging and addressing factors that contribute to differential outcomes among groups of students are critical to ensuring that all students routinely have opportunities to experience high-quality mathematics instruction, learn challenging mathematics content and receive the support necessary to be successful. Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement (NCTM, 2014).

As we have argued, mathematics has cognitive, sociocultural, and socio-linguistic dimensions, and equitable mathematics instruction must integrate these strands of proficiency, practice, and discourse. Leveraging the strengths that diverse students bring to the classroom, and integrating their experiences into math lessons, can support equitable approaches for teaching mathematics. To make math education the "great equalizer," as Horace Mann once envisioned, we must engage all students in mathematical discourse, and empower all students with literacy in the language of mathematics.

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